Counting, Continued (Ch. 13):

- Permutations
  - Definition: order or arrangement

- Example: 8 runners in a race
  - $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$
    This is the number of ways runners could finish the race.
  - If we only care about 1st, 2nd, and 3rd, we only multiply first 3:
    $8 \cdot 7 \cdot 6 = 336$
  "permutations of 8 things taken 3 at a time"

- n factorial $\rightarrow n!$
  - multiply numbers together starting with given number until reaching 1
  - MATH → PRB → 4: !
  - $\varnothing! = 1$

- Back to permutations:
  - Notation for 8 things taken 3 at a time → ${}_8P_3$
  - Permutations of n objects r at a time

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

  - Example:

$$ {}_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 8 \cdot 7 \cdot 6 = 336 $$

- Combinations
  - When we don't care about order, use combinations
  - Combinations of n objects chosen r at a time

$$_nC_r = \frac{n!}{r!(n-r)!}$$

- Back to example:

$$_8C_3 = \frac{8!}{3!(8-3)!}$$

$$= \frac{8!}{3!\,5!}$$

$$= \frac{8 \cdot 7 \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{1} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$$

$$= 56$$

- Calculator
  - Permutations

    n, MATH → PRB → 2: nPr, r

  - Combinations

    n, MATH → PRB → 3: nCr, r

- When do I use permutations or combinations?
  - Do I care what order things happen?
    If yes, use permutations.
  - Or, do I just care which ones are chosen?
    Use combinations.

- Models for Probability:
  - When sample space consists of equally likely outcomes, $P(\text{event}) = \dfrac{\#\text{ ways event}}{\#\text{ possible outcomes}}$
  - We don't have to list all possible outcomes. We can use permutations, combinations, or Fundamental Counting Principle.